Exam. Code : 103201
Subject Code : 1026

B.A./B.Sc. Semester—I

MATHEMATICS

Paper–I (Algebra)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **five** questions, selecting at least **two** from each section. All questions carry equal marks.

SECTION—A

1. (a) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & -7 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

(b) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$. Find non-singular matrices P and Q such that PAQ is in the normal form and hence determine the rank of A.

2. (a) If A, B are two n-rowed square matrices, then show that :

$$\rho(A)+\rho(B) - n \le \rho(AB) \le \min\,[\rho(A), \rho(B)]$$

(b) Determine whether the following matrices have same column space or not :

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 9 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 15 \end{bmatrix}$$

3. (a) Investigate for what values of a, b the following equations :

$x - 2y + 3z = 1$, $x + y - z = 4$, $2x - 2y + az = b$ have :

(i) no solution

(ii) unique solution

(iii) an infinite number of solutions.

(b) Prove that if the eigen values of A are $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ then the eigen values of A^2 are $\lambda_1^2, \lambda_2^2, \ldots\ldots \lambda_n^2$.

4. (a) Find the characteristic roots and the associated characteristic vectors for the matrix :

$$\begin{bmatrix} -3 & -9 & -12 \\ 1 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Verify Cayley–Hamilton theorem for the matrix A, where :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

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5. (a) Find the characteristic and minimal equation of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(b) Write the quadratic form of the symmetric matrix :

$$\begin{bmatrix} o & a & b & c \\ a & o & \ell & m \\ b & \ell & o & p \\ c & m & p & o \end{bmatrix}$$

SECTION—B

6. (a) Classify the following form as definite, semi-definite and indefinite :

$$2x^2 + 2y^2 + 3z^2 - 4yz - 4zx + 2xy.$$

(b) Solve the equation $x^3 - 7x^2 + 36 = 0$, one root being double the other.

7. (a) Solve the equation $x^4 - 8x^3 + 23x^2 - 28x + 12 = 0$, it being given that the difference of two of the roots is equal to other difference of the other two.

(b) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ may be in H.P.

8. (a) Diminish the root of the equation :

$a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ by h and find the condition that the second and third terms may be removed simultaneously.

(b) If α, β, γ are the roots of the equation :

$x^3 - 5x^2 + x + 12 = 0$, find the value of $\sum \alpha^2 (\beta + \gamma)$.

9. (a) Use Cardan's method to solve :

$x^3 + x^2 - 16x + 20 = 0$.

(b) Solve by Descarte's Method :

$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

10. (a) Show that the equation $x^8 - x^3 + x^2 - x + 1 = 0$ must have at least 4 non-real roots.

(b) Find by Newton's method of approximation the positive roots of $x^3 - 2x - 5 = 0$.

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